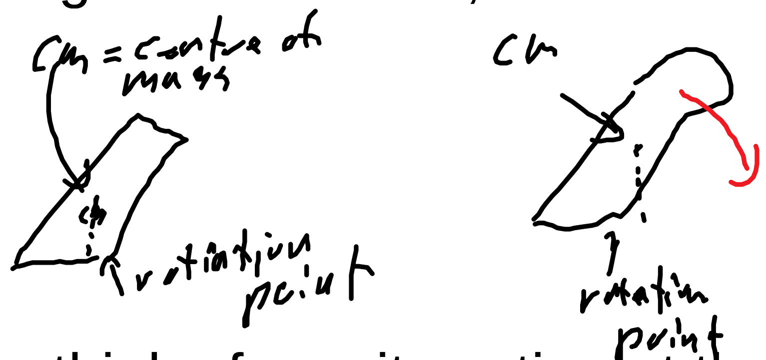
Wednesday, February 01, 2017 8:45 AM

Centre of Mass/Centre of Gravity - Torque

hand in labs
look at toys
balancing masses
equation

Leaning tower - add the extra part, it falls because

- think seesaw - teetertotter - playground lever
you have a fulcrum - rotation point - if there is more weight on one side, it will fall that way.



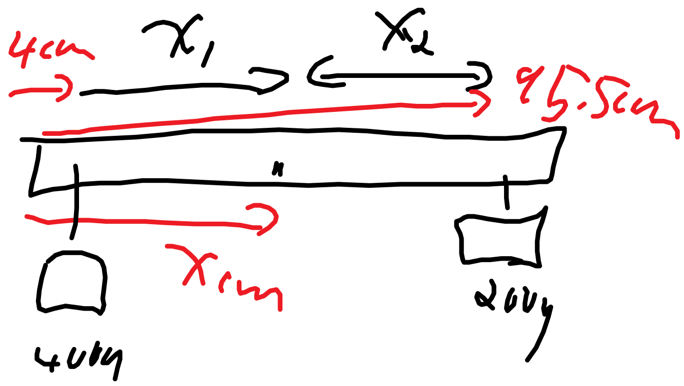
You can think of gravity acting at the centre of mass, sometimes called the centre of gravity.

Horsey guy - balanced when you have a weight.

the weight makes the centre of gravity below the rotation point, stabilizing the system. Without the weight, the centre of gravity is higher and to the side of the base.

Look at a ruler with 400g at 4.0 cm and 200 g at 95.5 cm will balance at what point? (centre of

mass/gravity)
Shirley - mass x distance on both sides should be equal.



One method

$$400x_1 = 200x_2$$

$$x_1 + x_2 = 95.5 - 4$$

$$400x_1 = 200(91.4 - x_1)$$

$$400x_1 = 18280 - 200x_1$$

$$600x_1 = 18280$$

$$x_1 = 30.47$$

$$30.47 + 4 = \boxed{34.47\text{cm}}$$

Centre of mass equation
$m_{total}x_{cm} = m_1x_1 + m_2x_2 + ... + m_nx_n$
(can derive from work= Fd = mgd where d is proportional to x)

$(200g+400g)\ x_{cm} = 200g\ (95.5cm) + 400g(4.0cm)$

$600x = 19{,}100 + 1600 = 20700$

$x = 20700/600 = \boxed{34.5}$ cm same! wow!

p170-171

problems 39, 45-51 odds

define torque

volume of sphere = $4/3\ \pi r^3$

$d = 1.13 \times 10^{-10}$

m=16 O

m=12 C

reference point

x=0

$m_t\, x_{cm} = m_1 x_1 + m_2 x_2$

$(16+12)\, x_{cm} = 16 \times 0 + 12(1.13 \times 10^{-10})$

$$x_{cm} = \frac{12\,(1.13 \times 10^{-10})}{28}$$

Torque, $\tau = F \times r$ ← radial distance

Greek letter Tau

Force

vector cross product

$$\boxed{\tau = Fr\sin\theta}$$

$Fr\sin\theta = \tau$

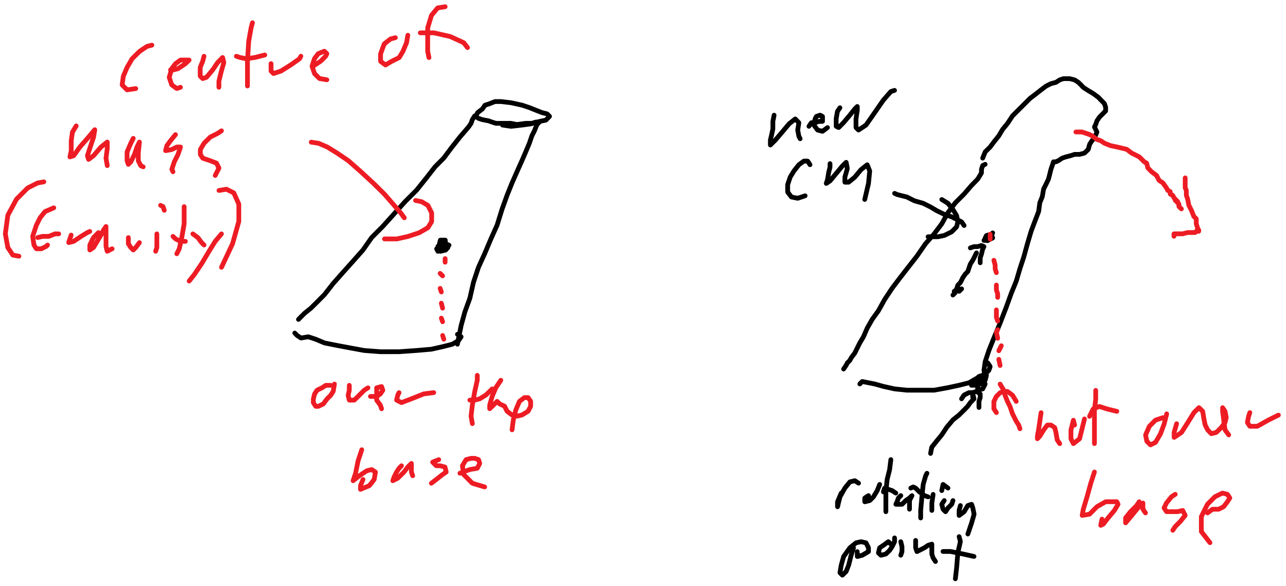
units are Nm (Not Joule)

Big Idea:
rotational equilibrium - sum of all torques = 0
or the clockwise torques = counter-clockwise torques
when static or no rotational acceleration

translational equilibrium - sum of all forces = 0
no translational acceleration

static equilibrium - both rotational and translational equilibrium

Toys
tower of Pisa -



Adding the extra top moves the centre of mass so that it is not over the base, this results in a force

past the rotation point(edge of the base) and it falls over.

horsey guy - with the weight, the centre of mass is beneath the base, so it is stable.

without the weight, the centre of mass is above but can easily shift to the side (small base).

Look at a metre stick with a 400g mass at 10cm and a 200g mass at 80 cm. Where will it balance?(the centre of mass or gravity)

Big Idea - the centre of mass is **<u>not</u>** the place with the same mass on both sides.

Derive equation - W=Fd, if energy is conserved, then Fd on one side = Fd on the other side if the masses moved up/down a little.
The distance they move is proportional to the distance to the rotation point.

So if $F_1d_1 = F_2d_2$ and d is proportional to x
$m_1gx_1 = m_2gx_2$
or
$m_1x_1 = m_2x_2$
d is the displacement of the weight
x is the distance to the reference point (here is the rotation point)

distance between the masses = x_1+x_2
x_2= 70cm - x_1
400g x_1 = 200g (70cm - x_1)
2 x_1 = 70cm - x_1
3 x_1 =70cm
x_1 = 23cm
so it should balance at 33cm (pretty close)

general equation for the centre of mass:

$$\boxed{m_{total}x_{cm} = m_1x_1+m_2x_2 +...+m_nx_n}$$

x_{cm} is the position of the centre of mass.

Alternate solution

(400g+200g) x_{cm} = 400g (10cm) + 200g (80cm)

x_{cm} = (4000+16000)/600=33.3333
$\boxed{\text{33cm}}$ same answer! wow!

p170-171
Q39, 45-51 odds
Bring labbook pre-read p41-46

15minutes we will go over p39, introduce torque

$1.13\times10^{-10}\,m$

reference point

$$m_T\, x_{cm} = m_1 x_1 + m_2 x_2$$

$$(16+12)\, x_{cm} = 16(0) + 12(1.13\times10^{-10}\,m)$$

$$x_{cm} = \frac{12(1.13\times10^{-10}\,m)}{28}$$

45 $V=\frac{4}{3}\pi r^3$

Look at a metre stick with a 400g mass at 10cm and a 200g mass at 80 cm. Where will it balance?(the centre of mass or gravity)

Big Idea - the centre of mass is **<u>not</u>** the place with the same mass on both sides.

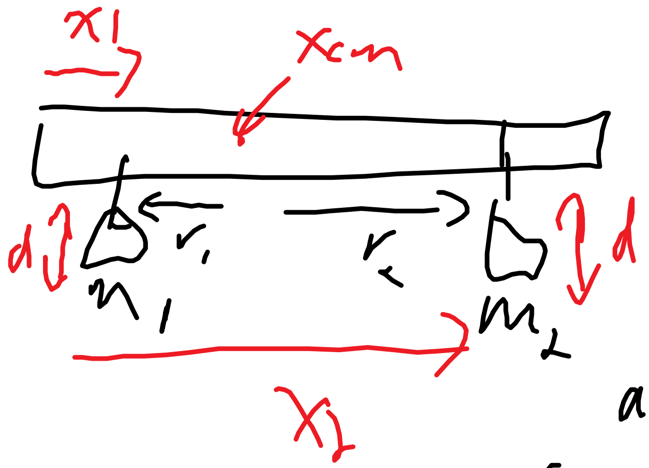
The centre of mass is where the mass distribution through space is balanced.
mass x position

Why does that work?

derive it from conservation of energy - work is Fd
F=mg

mgd   where d is the distance the mass moves up/down

but d is proportional to the position, x, distance to the rotation point.



about the cm

since $m_1 g d_1 = m_2 g d_2$

and $d_1 \propto r_1$

$d_2 \propto r_2$

$$m_1 r_1 = m_2 r_2$$

$$r_1 = x_{cm} - x_1$$

$$r_2 = x_2 - x_{cm}$$

$$m_1 (x_{cm} - x_1) = m_2 (x_2 - x_{cm})$$

$m_1 x_{cm} - m_1 x_1 = m_2 x_2 - m_2 x_{cm}$

$m_1 x_{cm} + m_2 x_{cm} = m_1 x_1 + m_2 x_2$

$$\boxed{(m_1 + m_2) x_{cm} = m_1 x_1 + m_2 x_2}$$

(400g + 200g) x_{cm} = 400g (10cm) + 200g (80cm)

x_{cm} = (4000+16000)/600=33.3333
x_{cm} = 33 cm

or $m_1r_1 = m_2r_2$ and r_1+r_2=70cm

400g(r_1) = 200g (70-r_1)
2r_1 = 70 - r_1
3r_1 = 70
r_1 = 70/3=23.3333
so x_{cm}=23.33 + 10=33.33cm
=33cm

p170-171
problems 39, (15 minutes) 45-51
45 need volume of a sphere = 4/3 πr^3